Tangents and Normals

Chapter 6

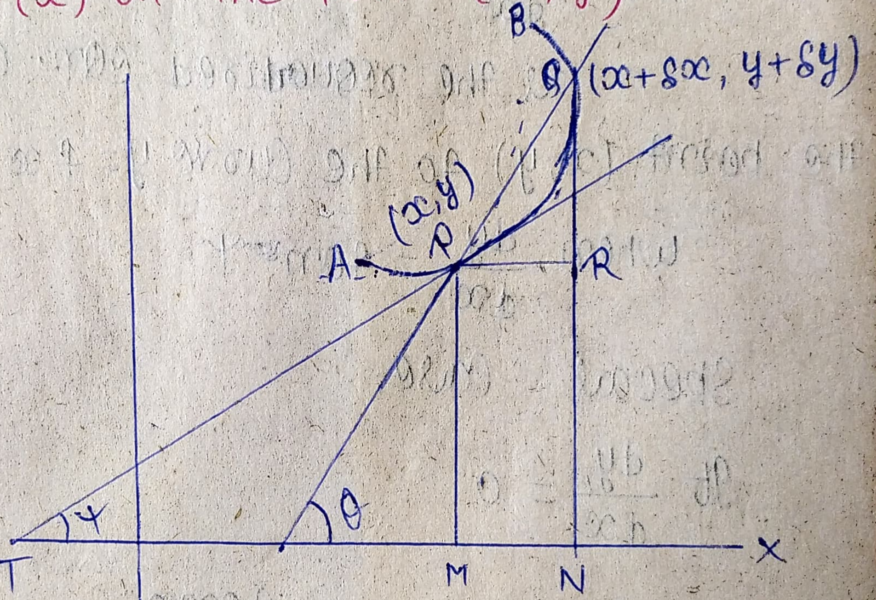
Theorm:- ① Prove that the equation to the tangent to the curve $y=f(x)$ at (x,y) is $Y-y=\frac{dy}{dx}(X-x)$ and that to the curve $f(x,y)=0$ is

$$(X-x)f_x+(Y-y)f_y=0.$$

(or)

To find the equation of tangent to the curve $y=f(x)$ at the point (x,y).

Ans.→



Let AB be the equation of a curve $y=f(x)$ — ①

we want to know the equation of tangent at the pt $P(x,y)$ to the curve ②

Let Q be $(x+\delta x, y+\delta y)$ another point on the curve very close to P

Let (X,Y) be the current co-ordinate Then the eqn. of PQ is given by

$$Y-y=\frac{y+\delta y-y}{x+\delta x-x}(X-x)$$

$Y - y = \frac{\delta y}{\delta x}(X - x)$ —— ③

when $Q \to P$, then $\delta x \to 0$ and the line PQ is tangent ~~to~~ to the curve at the point P makes and away ~~the~~ $\tan \psi$.

Let the tangent at P, i.e. PT with x-axis

Hence, on taking limit when $\delta x \to 0$

$$Y - y = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}(X - x)$$

$$Y - y = \frac{dy}{dx}(X - x)$$

is the required eqn. of tangent at the point (x, y) to the curve $y = fx$

when, $\frac{dy}{dx} = \tan\psi$

Specail case

If $\frac{dy}{dx} = 0$

$\therefore \tan\psi = 0 = \tan 0$

$\psi = 0$

Hence the tangent is $\parallel$ to x axis

when $\psi = 90°$

tangent is $\parallel$ to y-axis.

$$Y - y = \frac{dy}{dx}(X - x)$$

$$Y - y = \frac{-fx}{fy}(X - x)$$

or, $(Y-y)f_y = -f_x(X-x)$

or, $f_x(X-x)+f_y(Y-y)$ is the required eqn. of tangent to the curve $f(x,y)=0$ at the point (x,y)

At the pt (x,y), to the curve $y=f(x)$

$$Y-y=\frac{dy}{dx}(X-x)$$

when the eqn. of the curve $f(x,y)=0$ the eqn. of tangent

$$(X-x)f_x+(Y-y)f_y=0$$

Prove that

(i) The cartesian sub ~~normal~~ tangent $= \frac{y}{y_1}$,

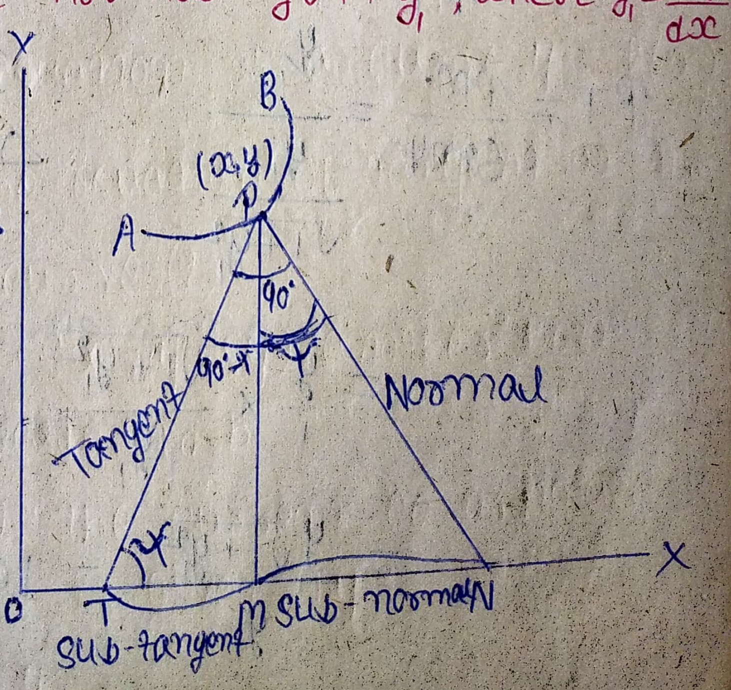
(ii) The cartesian sub-normal $= yy_1$

(iii) The Length of the tangent $= \frac{y}{y_1}\sqrt{1+y_1^2}$

(iv) The Length of the normal $= y\sqrt{1+y_1^2}$, where $y_1=\frac{dy}{dx}$.

Ans. → Let AB be a curve whose eqn. is $y=f(x)$

Let $P(x,y)$ be away point on the curve.



Hence PT = Tangent at pt. $P(x, y)$ to the curve

or, PN = Normal

Let tangent PT makes an angle ψ with the axis of X, then $\tan\psi = \frac{dy}{dx} = y_1$

Let the tangent at P meet the X-axis at T

from P draw PM ⊥ to OX

the TM = sub Tangent

MN = sub normal

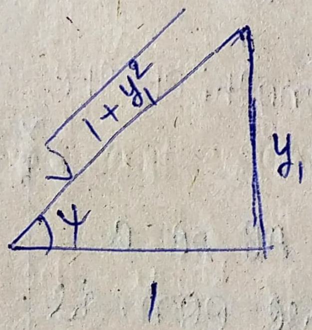
$\therefore OM = x$

$PM = y$

Length of tanget

In ΔPTM

$\sin\psi = \frac{PM}{PT}$



$PT = \frac{PM}{\sin\psi} = \frac{y_1}{\frac{y_1}{\sqrt{1+y_1^2}}}$

$= \frac{y}{1} \times \frac{\sqrt{1+y_1^2}}{y_1}$

$= \frac{y\sqrt{1+y_1^2}}{y_1}$ = Length of tangent

Length of Normal

In ΔPMN

$$\cos\psi = \frac{PM}{PN}$$

$$\therefore PN = \frac{PM}{\cos\psi} = \frac{\frac{y}{1}}{\frac{1}{\sqrt{1+y_1^2}}} = \frac{y}{1} \times \frac{\sqrt{1+y_1^2}}{1}$$

$$= y\sqrt{1+y_1^2}$$

Length of sub Tangent

In ΔPTM

$$\tan\psi = \frac{PM}{TM}$$

sub Tangent $TM = \frac{PM}{\tan\psi} = \frac{y}{y_1} = \frac{y}{\frac{dy}{dx}}$

Length of sub normal

In ΔPMN

$$\frac{\tan\psi}{1} = \frac{MN}{PM}$$

sub normal $MN = PM \tan\psi$

$$= y \times y_1$$